

IOWA
End-of-Course
Assessment
Programs
Released Items

MATRIX ALGEBRA

1 Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$. What is the

solution to the matrix equation $AX = B$?

A $X = \begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix}$

INCORRECT: The student found

$$A^{-1} = \frac{1}{3-4} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = -1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix},$$

then multiplied A^{-1} and B in the wrong order.

$$X = BA^{-1}$$

B $X = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$

INCORRECT: The student found the wrong

$$A^{-1} \text{ as } A^{-1} = 1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix},$$

then multiplied A^{-1} and B in the wrong order.

$$X = BA^{-1}$$

C $X = \begin{bmatrix} -6 & -5 \\ 4 & 3 \end{bmatrix}$

CORRECT: The student found

$$A^{-1} = \frac{1}{3-4} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = -1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix},$$

then multiplied A^{-1} and B in the correct order.

$$X = A^{-1}B$$

D $X = \begin{bmatrix} 6 & 5 \\ -4 & -3 \end{bmatrix}$

INCORRECT: The student found the wrong

$$A^{-1} \text{ as } A^{-1} = 1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix},$$

then multiplied A^{-1} by B .

$$X = A^{-1}B$$

CCSS Conceptual Category:

Algebra

CCSS Domain:

Reasoning with Equations and Inequalities

2 If matrix A has dimensions of 4×6 and matrix B has dimensions of 6×2 , what are the dimensions of matrix AB ?

A 2×4

INCORRECT: The student thought the dimensions of AB were the number of columns of B by the number of rows of A .

B 4×2

CORRECT: The student identified the dimensions of AB as the number of rows in A by the number of columns in B .

C 6×6

INCORRECT: The student thought the result should be a square matrix with the common dimensions of A and B .

D 24×12

INCORRECT: The student thought the dimensions were the products of the row and the columns: $(4)(6) \times (6)(2)$.

CCSS Conceptual Category:

Algebra

CCSS Domain:

Reasoning with Equations and Inequalities

- 3** A volleyball tournament was played over a two-day period. Ticket prices were \$6 for adults and \$4 for students. Matrix T shows the numbers of tickets sold for the tournament and matrix C shows the ticket prices in dollars.

$$T = \begin{array}{c} \text{Adult} \quad \text{Student} \\ \text{Day 1} \begin{bmatrix} 200 & 325 \end{bmatrix} \\ \text{Day 2} \begin{bmatrix} 250 & 400 \end{bmatrix} \end{array}, \quad C = \begin{array}{c} \text{Adult} \\ \text{Student} \end{array} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

Which matrix operation gives the total revenue, in dollars, from ticket sales for each day of the tournament?

A $C \times T$

INCORRECT: The student did not take into account the dimensions of each matrix when determining the order of the product.

B $T \times C$

CORRECT: The student multiplied the number of tickets sold (2×2 matrix) by the ticket price (2×1 matrix).

C $2T \times C$

INCORRECT: The student tried to account for the 2-day event by multiplying by 2.

D $2(C \times T)$

INCORRECT: The student used the product (ticket price) \times (number of tickets sold), and then tried to account for the 2-day event by multiplying that product by 2.

CCSS Conceptual Category:

Algebra

CCSS Domain:

Creating Equations

- 4 What should $\begin{bmatrix} 7 \\ 4 \end{bmatrix}$ be multiplied by to solve the matrix equation below?

$$\begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

A $\frac{1}{4} \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$

INCORRECT: The student found the determinant by adding, not subtracting, $\left(\frac{1}{(2)(3) + (-2)(1)}\right)$ and interchanged the non-leading diagonal elements.

B $\frac{1}{4} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$

INCORRECT: The student found the determinant by adding, not subtracting, $\left(\frac{1}{(2)(3) + (-2)(1)}\right)$.

C $\frac{1}{8} \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$

INCORRECT: The student interchanged the non-leading diagonal elements.

D $\frac{1}{8} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$

CORRECT: The student found the determinant $\left(\frac{1}{(2)(3) - (-2)(1)}\right)$, interchanged the leading diagonal elements, and changed the signs of the non-leading diagonals.

CCSS Conceptual Category:

Algebra

CCSS Domain:

Reasoning with Equations and Inequalities

5 What should $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ be multiplied by to

obtain the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$?

A $\begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$

INCORRECT: The student found the inverse of the matrix.

B $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

CORRECT: The student used the identity matrix.

C $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

INCORRECT: The student placed the 1's on the wrong diagonal.

D $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

INCORRECT: The student thought the identity matrix has all elements of 1.

CCSS Conceptual Category:

Algebra

CCSS Domain:

Arithmetic with Polynomials and Rational Expressions